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Substituting in this equation, the values of x , y , and z , and reducing, we obtain

$$r(bc+ac+ab-a^2-b^2-c^2)=bc+ac+ab-a^2-b^2-c^2.$$

From this equation we find that $r=1$, unless

$$\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = 0 \dots (2).$$

But (2) is the condition that the triangle a , b , c is equilateral. Therefore, either $r=1$, or else both triangles are equilateral.

Also solved by *G. B. M. ZERR*, *J. W. YOUNG*, and *FRANK A. GRIFFIN*.

131. Proposed by *J. W. YOUNG*, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.

Prove that $\lambda + \mu\omega + \nu\omega^2$, where λ , μ , ν are integers whose sum is ± 1 , represents the points of a quilt formed by regular hexagons. ω =primitive cube root of unity. [From Harkness and Morley's *Introduction to Theory of Functions*.]

Solution by the PROPOSER.

$$\omega = -\frac{1}{2} + i(\frac{1}{2}\sqrt{3}), \omega = -\frac{1}{2} - i(\frac{1}{2}\sqrt{3}) \quad (i^2 = -1).$$

Then $\lambda + \mu\omega + \nu\omega^2 = \lambda - \frac{1}{2}\mu - \frac{1}{2}\nu + i(\mu - \nu)\sin\frac{1}{2}\pi$. Taking rectangular coördinates this quantity represents the points (x, y) , when

$$\left. \begin{aligned} x &= \lambda - \frac{1}{2}\mu - \frac{1}{2}\nu \\ y &= (\mu - \nu)\sin\frac{1}{2}\pi \end{aligned} \right\} \begin{aligned} &\lambda + \mu + \nu = \pm 1 \\ &(\lambda, \mu, \nu \text{ are integers}) \end{aligned}$$

$y = n\sin\frac{1}{2}\pi$, when $n = (\mu - \nu) = \text{any integer}$.

Then $\mu = n + \nu$.

Substituting in x and in $\lambda + \mu + \nu = \pm 1$, we obtain

$$\left. \begin{aligned} 2\lambda - 2\nu - n &= 2x \\ \lambda + 2\nu + n &= \pm 1 \end{aligned} \right\} \dots (1).$$

$$\therefore x = \frac{1}{2}(3\lambda \pm 1).$$

The points required are, then, those whose coördinates are $[\frac{1}{2}(3\lambda \pm 1, n\sin\frac{1}{2}\pi]$, where λ , n are any integers with the one restriction that when n is *odd*, λ is even, and *vice versa*. This restriction is evident, since (1) shows that $\lambda + n$ must be *odd*.

We have then following values of x and y :

A.	For $n=0, 2, 4, 6$, etc.						
	$\lambda=$	1	3	5	7	9	etc.
	$x=$	1 2	4 5	7 8	10 11	13 14	
	For $n=1, 3, 5, 7$, etc.						
	$\lambda=$	0	2	4	6	8	
	$x=$	$-\frac{1}{2} \frac{1}{2}$	$\frac{5}{2} \frac{7}{2}$	$\frac{11}{2} \frac{13}{2}$	$\frac{17}{2} \frac{19}{2}$	$\frac{23}{2} \frac{25}{2}$	

$$y = n \sin \frac{1}{2} \pi.$$

Similarly for negative values of n, λ .

MECHANICS.

96. Proposed by **GEORGE R. DEAN**, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Two particles, subject to their mutual attraction and that of a fixed center, move in a plane containing the center. Find the motion under the law of the inverse square.

Solution by **G. B. M. ZERR**, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Take the center of force as origin. Let m, m_1, m_2 be the masses of the center of force and particles, respectively. r, r_1, ρ the distances of the particles from the center of force and from each other, respectively. $(x, y), (x', y')$ the coördinates of the particles. The differential equations of motion of the two particles relative to the center of force are

$$\left. \begin{aligned} \frac{d^2 x}{dt^2} &= -\frac{m+m_1}{r^3}x + \frac{x'-x}{\rho^3}m_2 - \frac{m_2 x'}{r_1^3} \\ \frac{d^2 y}{dt^2} &= -\frac{m+m_1}{r^3}y + \frac{y'-y}{\rho^3}m_2 - \frac{m_2 y'}{r_1^3} \end{aligned} \right\} \dots (1).$$

$$\left. \begin{aligned} \frac{d^2 x'}{dt^2} &= -\frac{m+m_2}{r_1^3}x' + \frac{x-x'}{\rho^3}m_1 - \frac{m_1 x}{r^3} \\ \frac{d^2 y'}{dt^2} &= -\frac{m+m_2}{r_1^3}y' + \frac{y-y'}{\rho^3}m_1 - \frac{m_1 y}{r^3} \end{aligned} \right\} \dots (2).$$

Where $\rho = \sqrt{[(x'-x)^2 + (y'-y)^2]}$.

$$\text{Multiply (1) by } 2m_1 \frac{dx}{dt} - 2m_1 \frac{m_1 \frac{dx}{dt} + m_2 \frac{dx'}{dt}}{m+m_1+m_2}.$$

$$2m_1 \frac{dy}{dt} - 2m_1 \frac{m_1 \frac{dy}{dt} + m_2 \frac{dy'}{dt}}{m+m_1+m_2}.$$